# Numerical Methods Lecture XIV: Simulated Estimation <br> (See Keane and Moffitt 1996) 

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## Statutory Marginal Tax Rates-1



## Implicit Marginal Tax Rates-1

## THE DEAD ZONE

Earned Income less Social Security, Federal and State Income Tax plus EITC, Food Stamps, Medicaid/SCHIP, Section 8 Housing (line) VERSUS Welfare cash grant + subsidies (dot)

Hypothetical Virginia Family of 3


## Implicit Marginal Tax Rates-2

## Effective Marginal Tax Rates for a Head of Household with Two Children



Average effective marginal tax rates facing a single parent with two children living in Colorado. The effective marginal rate is the the marginal tax rate faced in the formal tax system (federal, state, and payroll) in addition to the rates arising from the reduction in disposable income from the loss of transfer benefits. The tax rules used for federal and state income taxes are for CY2011. The payroll tax rate does not include the temporary reduction of the employee portion of the tax. Hypothetical exchange subsidy values were calculated to display the eventual impact of the Affordable Care Act for a worker without employer provided coverage based on C8O estimates discounted back to 2011.

## Implicit Marginal Tax Rates-3

## Case 2: Maximum Available Tax and Benefit Programs

 (Single Parent with Two Children in Colorado, 2011)

## The Problem is complicated!

- We spend trillions on transfer programs
- Implicit marginal tax rates frequently bigger deal than statutory tax rates
- Now, if you're on one program you're on a lot of them
- In Keane and Moffitt (circa 1989)
- $89 \%$ of AFDC recipients were on Medicaid and Food Stamps
- $42 \%$ of AFDC recipients were on some fourth program (like housing)
- Enormous implicit marginal tax rates interact


## Keane \& Moffitt

- Look at female heads
- AFDC, Food stamps, housing, and labor supply
- Produce four-equation model
- Simulate outcomes given parameters
- Estimate parameters


## Illustrative Cumulative Tax Rates: CA

| Weekly Income |  |  | Tax Rate from $\mathrm{H}=0$ to $\mathrm{H}=20$ | Tax Rate from $\mathrm{H}=20$ to $\mathrm{H}=40$ |
| :---: | :---: | :---: | :---: | :---: |
| $H=0$ | $H=20$ | $H=40$ |  |  |
| 0 | 104 | 208 | . | . |
| 124 | 30 | 0 | 0.90 | 0.29 |
| 16 | 16 | 0 | 0 | 0.15 |
| 138 | 132 | 107 | 0.06 | 0.24 |
| 0 | -8 | -26 | 0.08 | 0.17 |
| 0 | -21 | -21 | 0.20 | 0 |
| 278 | 253 | 268 | . | . |
| . | . | . | 1.24 | 0.86 |

## Illustrative Cumulative Tax Rates: MN

|  | Weekly Income |  |  | Tax Rate <br> from <br> $\mathrm{H}=0$ <br> $\mathrm{H}=20$ | to |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  | $H=0$ | $H=20$ | $H=40$ |  |  |
| from |  |  |  |  |  |
| $\mathrm{H}=20$ |  |  |  |  |  |
| $\mathrm{H}=40$ |  |  |  |  |  | to

## Illustrative Cumulative Tax Rates: OH

| Weekly Income |  |  | Tax Rate from <br> $\mathrm{H}=0$ to $\mathrm{H}=20$ | Tax Rate from <br> $\mathrm{H}=20$ to <br> $\mathrm{H}=40$ |
| :---: | :---: | :---: | :---: | :---: |
| $H=0$ | $H=20$ | $H=40$ |  |  |
| 0 | 104 | 208 |  | $\cdot$ |
| 60 | 0 | 0 | 0.58 | 0 |
| 44 | 30 | 4 | 0.13 | 0.29 |
| 87 | 71 | 37 | 0.15 | 0.33 |
| 0 | -8 | -26 | 0.08 | 0.17 |
| 0 | -21 | -21 | 0.20 | 0 |
| 191 | 176 | 202 |  |  |
| . |  |  | 1.14 | 0.75 |

## Illustrative Cumulative Tax Rates: KS

| Weekly Income | Tax Rate <br> from | Tax Rate <br> from |
| :--- | :--- | :--- | :--- |
| $H=0$ | $H=20$ | $H=40$ |
|  | $H=0$ <br> $H=20$ | $H=20$ <br> $H=40$ |

Kansas

Earnings AFDC
Food Stamp
Housing
Taxes
Work Expns.
Net Income
Cumulative
Tax Rate

0
76
38
68
0
0
82
104
0
31
64
-8
-21
170

208
0
0
31
-26
-21
192
.
1.12
0.79

## Keane \& Moffitt: Utility

- $U$ is:

$$
U\left(H, Y, P_{1}, P_{2}, P_{3}\right)=\overline{U(H, Y)}-\psi_{1} P_{1}-\psi_{2} P_{2}-\psi_{3} P_{3}
$$

- Where $H$ is hours of work.
- Y is disposable income.
- $P_{j}$ is a dummy variable for participation in program $j$.
- $\psi_{j}$ is the marginal disutility of participating in program $j$.
- Limit $H \in\{0,20,40\}$. Limits to $3 \cdot 2^{3}=24$ possibilities.


## Keane \& Moffitt: Budget Constraint

- Disposable income is defined as:

$$
Y\left(H, P_{1}, P_{2}, P_{3}\right)=w H+N+P_{1} B_{1}(H)+P_{2} B_{2}(H)+P_{3} B_{3}(H)-T(H)
$$

- Where $w$ is the hourly wage rate
- $N$ is nontransfer nonlabor income
- $B_{j}(H)$ is benefit function for program $j$.
- $T(H)$ is the tax function.
- In estimation, use $\gamma$ to capture benefit cash value.


## Keane \& Moffitt: Optimization

- Households choose from three choices of hours and 8 choices of program participation
- All interact nonlinearly with income
- Choose the best of all activities. Choose $j$ iff:

$$
U_{j} \geq U_{k} \quad \forall k \in\{1,2, \ldots, 24\}
$$

## Keane \& Moffitt: Take it to the data!

- Assume a form of utility:

$$
\begin{aligned}
U\left(H, Y, P_{1}, P_{2}, P_{3}\right)= & \alpha H+Y-\beta_{H H} H^{2}-\beta_{Y Y} Y^{2}+\beta_{H Y} H Y \\
& -\psi_{1} P_{1}-\psi_{2} P_{2}-\psi_{3} P_{3} \\
& +\phi_{12} P_{1} P_{2}+\phi_{13} P_{1} P_{3}+\phi_{23} P_{2} P_{3} \\
& -\delta_{1} H P_{1}-\delta_{2} H P_{2}-\delta_{3} H P_{3} \\
& -\eta_{1} Y P_{1}-\eta_{2} Y P_{2}-\eta_{3} Y P_{3}
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\end{aligned}
$$

- Ordinary utility from hours and income


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\end{aligned}
$$

- Ordinary utility from hours and income
- Direct disutility from participation


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& -\delta_{1} H P_{1}-\delta_{2} H P_{2}-\delta_{3} H P_{3} \\
& -\eta_{1} Y P_{1}-\eta_{2} Y P_{2}-\eta_{3} Y P_{3}
\end{aligned}
$$

- Ordinary utility from hours and income
- Direct disutility from participation
- Interactions from multiple participation


## Keane \& Moffitt: Take it to the data!

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$$
\begin{aligned}
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& -\psi_{1} P_{1}-\psi_{2} P_{2}-\psi_{3} P_{3} \\
& +\phi_{12} P_{1} P_{2}+\phi_{13} P_{1} P_{3}+\phi_{23} P_{2} P_{3} \\
& -\delta_{1} H P_{1}-\delta_{2} H P_{2}-\delta_{3} H P_{3} \\
& -\eta_{1} Y P_{1}-\eta_{2} Y P_{2}-\eta_{3} Y P_{3}
\end{aligned}
$$

- Ordinary utility from hours and income
- Direct disutility from participation
- Interactions from multiple participation
- Interaction of program on hours


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$$
\begin{aligned}
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& -\psi_{1} P_{1}-\psi_{2} P_{2}-\psi_{3} P_{3} \\
& +\phi_{12} P_{1} P_{2}+\phi_{13} P_{1} P_{3}+\phi_{23} P_{2} P_{3} \\
& -\delta_{1} H P_{1}-\delta_{2} H P_{2}-\delta_{3} H P_{3} \\
& -\eta_{1} Y P_{1}-\eta_{2} Y P_{2}-\eta_{3} Y P_{3}
\end{aligned}
$$

- Ordinary utility from hours and income
- Direct disutility from participation
- Interactions from multiple participation
- Interaction of program on hours
- Interaction of program on income


## Keane \& Moffitt: An issue(?)

$$
\begin{aligned}
U\left(H, Y, P_{1}, P_{2}, P_{3}\right)= & \alpha H+Y-\beta_{H H} H^{2}-\beta_{Y Y} Y^{2}+\beta_{H Y} H Y \\
& -\psi_{1} P_{1}-\psi_{2} P_{2}-\psi_{3} P_{3} \\
& +\phi_{12} P_{1} P_{2}+\phi_{13} P_{1} P_{3}+\phi_{23} P_{2} P_{3} \\
& -\delta_{1} H P_{1}-\delta_{2} H P_{2}-\delta_{3} H P_{3} \\
& -\eta_{1} Y P_{1}-\eta_{2} Y P_{2}-\eta_{3} Y P_{3}
\end{aligned}
$$

- Why doesn't $Y$ have a coefficient?
- There's an issue...what is it?
- Hint:
- Allow $\alpha$ and $\psi_{1}, \psi_{2}$, and $\psi_{3}$ to vary in the population:

$$
\begin{aligned}
\alpha & =X \bar{\alpha}+\epsilon_{\alpha} \\
\psi_{1} & =X \overline{\psi_{1}}+\epsilon_{\psi_{1}} \\
\psi_{2} & =X \overline{\psi_{2}}+\epsilon_{\psi_{2}} \\
\psi_{3} & =X \overline{\psi_{3}}+\epsilon_{\psi_{3}}
\end{aligned}
$$

- Assume $\epsilon_{\alpha}, \epsilon_{A}, \epsilon_{F}, \epsilon_{R}, \epsilon_{W}$ are multivariate normal with unrestricted covariance matrix


## Keane \& Moffitt: One final issue

- Wages for nonworkers are unobserved
- Specify wages as:

$$
\log (w)=X_{\nu}+\epsilon_{W}
$$

- How should they estimate this?


## Keane \& Moffitt: One final issue

- Wages for nonworkers are unobserved
- Specify wages as:

$$
\log (w)=X \nu+\epsilon_{W}
$$

- How should they estimate this?
- Two ways:
- Could do it beforehand
- Could do it along with the model


## Keane \& Moffitt: Estimation

- Say we knew the wages
- Given a set of parameters $\Theta=$ $\left\{\alpha, \sigma_{\alpha}, \sigma_{A}, \sigma_{F}, \sigma_{R}, \sigma_{W}, \rho_{\alpha A}, \rho_{\alpha F}, \rho_{\alpha R}, \rho_{\alpha W}, \rho_{A F}, \rho_{A R}, \rho_{A W}, \rho_{F R}\right.$, $\left.\rho_{F W}, \rho_{R W}, \overline{\psi_{1}}, \overline{\psi_{2}}, \overline{\psi_{3}}, \phi_{12}, \phi_{13}, \phi_{23}, \delta_{1}, \delta_{2}, \delta_{3}, \eta_{1}, \eta_{2} \eta_{3}\right\}$ and $X$, we can simulate the distribution and solve everyone's problem.
- They also make some things dependent on $X$, adding covariates to estimate.
- From that, we can write, for each person,

$$
P(j \mid X, \Theta)
$$

- From that we can produce a simulated likelihood and estimate.
- Alternatively, could write down the probabilities and likelihoods and use method of moments


## Keane \& Moffitt: Dealing with Wages

- Because wages are unobserved by econometrician but known by the individual, assuming we know it is wrong
- Keane and Moffit spend a long time on this
- The problem comes from the fact that our wage tells us about working and (not) working tells us about the wage
- Keane and Moffitt "integrate the wage out": take a number of random draws conditional on observables and take their average
- They also add a random error term to all utilities to make things smoother
- I'm not going to worry about these here


## DATA

| A | $F$ |  | Labor Supply |  |  | Row <br> Totals |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $R$ | Nonworkers | Part-Time | Full-time |  |
| 0 | 0 | 0 | 76 | 57 | 383 | 516 |
| 1 | 0 | 0 | 9 | 1 | 7 | 17 |
| 0 | 1 | 0 | 36 | 20 | 32 | 88 |
| 1 | 1 | 0 | 162 | 11 | 2 | 175 |
| 0 | 0 | 1 | 10 | 6 | 46 | 62 |
| 1 | 0 | 1 | 3 | 0 | 0 | 3 |
| 0 | 1 | 1 | 14 | 4 | 9 | 27 |
| 1 | 1 | 1 | 77 | 2 | 1 | 80 |
|  | tal |  | 387 | 101 | 480 | 968 |

# Results: Estimation 

Look at Table 2.

## Results: Interpretation

- $\beta_{H} H$ and $\beta_{Y} Y$ give wage and income elasticities
- Uncompensated: 1.82
- Income elasticity: -0.21
- Big disutilities from participation in everything but housing
- Not big interactive disutilities
- Cash value of housing: $\$ 0.10$
- Cash value of Medicaid: $\$ 0.48$
- Cash value of private health insurance $\phi: 0.62$


## Results: Alter the budget constraint

- Increasing eligibility phase out (reducing tax rate) for AFDC:
- Doesn't really impact labor
- Increases participation
- Wage shifts decrease participation and increases labor significantly


## External Validity

- Test against AFDC tax rate change in 1981
- See Table 7


## TAKEAWAYS

- Test against AFDC tax rate change in 1981
- See Table 7

